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Simplified thermodynamic model of gas turbine engine components

1 Constants

Table 1 — Constants for simplified thermodynamic calculation of a gas turbine engine

Symbol	Value	Unit of measure	Description
$c_{\mathrm{p}.a}$	1,005	$\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}$	Isobaric heat capacity of air
k_a	1,4	-	Ratio of specific heats of air
R_a	0,287	$\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}$	Gas constant of air
$c_{\mathrm{p}.cg}$	1,159	$\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}$	Isobaric heat capacity of combustion gas
k_{cg}	1,33	-	Ratio of specific heats of combustion gas
R_{cg}	0,2875	$\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}$	Gas constant of combustion gas
H_{n}	42 900	$\frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}$	Net calorific value of kerosine

2 Atmosphere

Table 2 — Input data

Symbol	Unit of measure	Description
M_f	-	Flight Mach number
T_1	K	Ambient temperature
p_1	kPa	Ambient pressure

Ambient density:

$$\rho_1 = \frac{p_1}{R_a \cdot T_1} \quad , \quad \frac{\text{kg}}{\text{m}^3} \quad .$$

Flight velocity:

$$V_f = \sqrt{k_a \cdot 1000 \cdot R_a \cdot T_1} \cdot M_f \quad , \quad \frac{\text{m}}{\text{s}} \quad ;$$

$$V_{f.h} = 3.6 \cdot V_f \quad , \quad \frac{\text{km}}{\text{h}} \quad .$$

3 Air intake

Table 3 — Input data

Symbol	Unit of measure	Description
M_f	-	Flight Mach number
T_1	K	Ambient temperature
p_1	kPa	Ambient pressure
σ_{intake}	-	Intake total pressure loss

Total temperature of atmospheric airflow:

$$T_1^* = T_1 \cdot \left(1 + \frac{k_a - 1}{2} \cdot M_f^2\right) \quad , \quad K \quad .$$

Total pressure of atmospheric airflow:

$$p_1^* = p_1 \cdot \left(1 + \frac{k_a - 1}{2} \cdot M_f^2\right)^{\frac{k_a}{k_a - 1}} \quad , \quad kPa \quad .$$

Total temperature at discharge:

$$T_2^* = T_1^* \quad , \quad K \quad .$$

Total pressure at discharge:

$$p_2^* = p_1^* \cdot \sigma_{intake} \quad , \quad kPa \quad .$$

4 Compressor

Table 4 — Input data

Symbol	Unit of measure	Description
T_2^*	K	Total temperature at entrance
p_2^*	kPa	Total pressure at entrance
π_c^*	-	Compressor pressure ratio
η_c^*	-	Compressor efficiency

Compressor work:

$$L_c = c_{pa} \cdot T_2^* \cdot (\pi_c^{* \frac{k_a - 1}{k_a}} - 1) \cdot \frac{1}{\eta_c^*} \quad , \quad \frac{kJ}{kg} \quad .$$

Total temperature at discharge:

$$T_3^* = T_2^* + \frac{L_c}{c_{pa}} \quad , \quad K \quad .$$

Total pressure at discharge:

$$p_3^* = p_2^* \cdot \pi_c^* \quad , \quad kPa \quad .$$

5 Fan

Table 5 — Input data

Symbol	Unit of measure	Description
T_2^*	K	Total temperature at entrance
p_2^*	kPa	Total pressure at entrance
m	-	Bypass ratio
π_{lpc}^*	-	Fan pressure ratio at primary flowpath
η_{lpc}^*	-	Fan efficiency at primary flowpath
π_{fII}^*	-	Fan pressure ratio at secondary flowpath
η_{fII}^*	-	Fan efficiency at secondary flowpath

Fan work at primary flowpath:

$$L_{\mathrm{lpc}}=c_{\mathrm{p}a}\cdot T_2^*\cdot\left(\pi_{\mathrm{lpc}}^{*\,\frac{k_a-1}{k_a}}-1\right)\cdot\frac{1}{\eta_{\mathrm{lpc}}^*}\quad,\quad\frac{\mathrm{kJ}}{\mathrm{kg}}\quad.$$

Total temperature at primary flowpath discharge:

$$T_3^*=T_2^*+\frac{L_{\mathrm{lpc}}}{c_{\mathrm{p}a}}\quad,\quad\mathrm{K}\quad.$$

Total pressure at primary flowpath discharge:

$$p_3^*=p_2^*\cdot\pi_{\mathrm{lpc}}^*\quad,\quad\mathrm{kPa}\quad.$$

Fan work at secondary flowpath:

$$L_{\mathrm{fII}}=c_{\mathrm{p}a}\cdot T_2^*\cdot\left(\pi_{\mathrm{fII}}^{*\,\frac{k_a-1}{k_a}}-1\right)\cdot\frac{1}{\eta_{\mathrm{fII}}^*}\quad,\quad\frac{\mathrm{kJ}}{\mathrm{kg}}\quad.$$

Total temperature at secondary flowpath discharge:

$$T_{3-\mathrm{II}}^*=T_2^*+\frac{L_{\mathrm{fII}}}{c_{\mathrm{p}a}}\quad,\quad\mathrm{K}\quad.$$

Total pressure at secondary flowpath discharge:

$$p_{3-\mathrm{II}}^*=p_2^*\cdot\pi_{\mathrm{fII}}^*\quad,\quad\mathrm{kPa}\quad.$$

6 Combustion chamber

Table 6 — Input data

Symbol	Unit of measure	Description
T_3^*	K	Total temperature at entrance
p_3^*	kPa	Total pressure at entrance
$g_{cooling.sb.hpt}$	-	Relative mass flow rate of cooling air for stationary blades of high pressure turbine
$g_{cooling.rb.hpt}$	-	Relative mass flow rate of cooling air for rotational blades of high pressure turbine
$g_{cooling.sb.ipt}$	-	Relative mass flow rate of cooling air for stationary blades of intermediate pressure turbine
$g_{cooling.rb.ipt}$	-	Relative mass flow rate of cooling air for rotational blades of intermediate pressure turbine
$g_{cooling.sb.lpt}$	-	Relative mass flow rate of cooling air for stationary blades of low pressure turbine
$g_{cooling.rb.lpt}$	-	Relative mass flow rate of cooling air for rotational blades of low pressure turbine
σ_{cc}	-	Combustion chamber total pressure loss
η_{cc}	-	Combustion efficiency
T_4^*	K	Total temperature at discharge

Relative fuel consumption:

$$q_f = \frac{c_{p.cg} \cdot T_4^* - c_{p.a} \cdot T_3^*}{H_n \cdot \eta_{cc}} \quad .$$

Total pressure at discharge:

$$p_4^* = p_3^* \cdot \sigma_{cc} \quad , \quad \text{kPa} \quad .$$

Relative mass flow rate at discharge:

$$\nu_4 = \left(1 - \sum_i g_{cooling.i}\right) \cdot (1 + q_f) \quad .$$

7 Turbine (compressor driving)

Table 7 — Input data

Symbol	Unit of measure	Description
T_4^*	K	Total temperature at entrance
p_4^*	kPa	Total pressure at entrance
ν_4	-	Relative mass flow rate at entrance
T_3^*	K	Total temperature of cooling air (at compressor discharge)
$g_{\mathrm{cooling.sb.t}}$	-	Relative mass flow rate of cooling air for stationary blades of the turbine
$g_{\mathrm{cooling.rb.t}}$	-	Relative mass flow rate of cooling air for rotational blades of the turbine
L_{c}	$\frac{\mathrm{kJ}}{\mathrm{kg}}$	Compressor work
η_m	-	Mechanical loss
η_{t}^*	-	Turbine efficiency

Total temperature at discharge of the first stationary blade row:

$$T_{\mathrm{sb.t}}^* = \frac{c_{\mathrm{p.}cg} \cdot T_4^* \cdot \nu_4 + c_{\mathrm{p.}a} \cdot T_3^* \cdot g_{\mathrm{cooling.sb.t}}}{c_{\mathrm{p.}cg} \cdot \left(\nu_4 + g_{\mathrm{cooling.sb.t}}\right)} \quad , \quad \mathrm{K} \quad .$$

Turbine work:

$$L_{\mathrm{t}} = \frac{L_{\mathrm{c}}}{\eta_m \cdot \left(\nu_4 + g_{\mathrm{cooling.sb.t}}\right)} \quad , \quad \frac{\mathrm{kJ}}{\mathrm{kg}} \quad .$$

Turbine pressure ratio:

$$\pi_{\mathrm{t}}^* = \left(1 - \frac{L_{\mathrm{t}}}{c_{\mathrm{p.}cg} \cdot T_{\mathrm{sb.t}}^* \cdot \eta_{\mathrm{t}}^*}\right)^{-\frac{k_{cg}}{k_{cg}-1}} \quad .$$

Total temperature at discharge of the last rotational blade row:

$$T_{\mathrm{rb.t}}^* = T_{\mathrm{sb.t}}^* - \frac{L_{\mathrm{t}}}{c_{\mathrm{p.}cg}} \quad , \quad \mathrm{K} \quad .$$

Total temperature at discharge:

$$T_5^* = \frac{c_{\mathrm{p.}cg} \cdot T_{\mathrm{rb.t}}^* \cdot \left(\nu_4 + g_{\mathrm{cooling.sb.t}}\right) + c_{\mathrm{p.}a} \cdot T_3^* \cdot g_{\mathrm{cooling.rb.t}}}{c_{\mathrm{p.}cg} \cdot \left(\nu_4 + g_{\mathrm{cooling.sb.t}} + g_{\mathrm{cooling.rb.t}}\right)} \quad , \quad \mathrm{K} \quad .$$

Total pressure at discharge:

$$p_5^*=\frac{p_4^*}{\pi_t^*} \quad , \quad \text{kPa} \quad .$$

Relative mass flow rate at discharge:

$$\nu_5=\nu_4+g_{\text{cooling.sb.t}}+g_{\text{cooling.rb.t}} \quad .$$

8 Turbine (fan driving)

Table 8 — Input data

Symbol	Unit of measure	Description
T_4^*	K	Total temperature at entrance
p_4^*	kPa	Total pressure at entrance
ν_4	-	Relative mass flow rate at entrance
T_3^*	K	Total temperature of cooling air (at compressor discharge)
$g_{\text{cooling.sb.lpt}}$	-	Relative mass flow rate of cooling air for stationary blades of the turbine
$g_{\text{cooling.rb.lpt}}$	-	Relative mass flow rate of cooling air for rotational blades of the turbine
m	-	Bypass ratio
L_{lpc}	$\frac{\text{kJ}}{\text{kg}}$	Fan work at primary flow
L_{fII}	$\frac{\text{kJ}}{\text{kg}}$	Fan work at secondary flow
$\eta_{m.\text{lp}}$	-	Mechanical loss
η_{lpt}^*	-	Turbine efficiency

Total temperature at discharge of the first stationary blade row:

$$T_{\text{sb.lpt}}^* = \frac{c_{\text{p}.cg} \cdot T_4^* \cdot \nu_4 + c_{\text{p}.a} \cdot T_3^* \cdot g_{\text{cooling.sb.lpt}}}{c_{\text{p}.cg} \cdot \left(\nu_4 + g_{\text{cooling.sb.lpt}}\right)} \quad , \quad \text{K} \quad .$$

Turbine work:

$$L_{\text{lpt}} = \frac{L_{\text{lpc}} + m \cdot L_{\text{fII}}}{\eta_{m.\text{lp}} \cdot \left(\nu_4 + g_{\text{cooling.sb.lpt}}\right)} \quad , \quad \frac{\text{kJ}}{\text{kg}} \quad .$$

Turbine pressure ratio:

$$\pi_{\text{lpt}}^* = \left(1 - \frac{L_{\text{lpt}}}{c_{\text{p}.cg} \cdot T_{\text{sb.lpt}}^* \cdot \eta_{\text{lpt}}^*}\right)^{-\frac{k_{cg}}{k_{cg}-1}} \quad .$$

Total temperature at discharge of the last rotational blade row:

$$T_{\text{rb.lpt}}^* = T_{\text{sb.lpt}}^* - \frac{L_{\text{lpt}}}{c_{\text{p}.cg}} \quad , \quad \text{K} \quad .$$

Total temperature at discharge:

$$T_5^* = \frac{c_{p.cg} \cdot T_{rb.lpt}^* \cdot \left(\nu_4 + g_{cooling.sb.lpt}\right) + c_{p.a} \cdot T_3^* \cdot g_{cooling.rb.lpt}}{c_{p.cg} \cdot \left(\nu_4 + g_{cooling.sb.lpt} + g_{cooling.rb.lpt}\right)} \quad , \quad K \quad .$$

Total pressure at discharge:

$$p_5^* = \frac{p_4^*}{\pi_{lpt}^*} \quad , \quad kPa \quad .$$

Relative mass flow rate at discharge:

$$\nu_5 = \nu_4 + g_{cooling.sb.lpt} + g_{cooling.rb.lpt} \quad .$$

9 Bypass duct

Table 9 — Input data

Symbol	Unit of measure	Description
T^*_{3-II}	K	Total temperature at entrance
p^*_{3-II}	kPa	Total pressure at entrance
σ_{II}	-	Duct total pressure loss

Total temperature at discharge:

$$T^*_{5-II} = T^*_{3-II} \quad , \quad \text{K} \quad .$$

Total pressure at discharge:

$$p^*_{5-II} = p^*_{3-II} \cdot \sigma_{II} \quad , \quad \text{kPa} \quad .$$

10 Mixer

Table 10 — Input data

Symbol	Unit of measure	Description
T_5^*	K	Total temperature at primary flow entrance
p_5^*	kPa	Total pressure at primary flow entrance
ν_5	-	Relative mass flow rate at primary flow entrance
$T_{5-\mathrm{II}}^*$	K	Total temperature at secondary flow entrance
$p_{5-\mathrm{II}}^*$	kPa	Total pressure at secondary flow entrance
m	-	Bypass ratio
σ_m	-	Mixer total pressure loss

Total temperature at discharge:

$$T_6^* = \frac{c_{\mathrm{p}.cg} \cdot T_5^* \cdot \nu_5 + c_{\mathrm{p}.a} \cdot T_{5-\mathrm{II}}^* \cdot m}{c_{\mathrm{p}.cg} \cdot \left(\nu_5 + m\right)} \quad , \quad \mathrm{K} \quad .$$

Total pressure at discharge:

$$p_6^* = \frac{p_5^* \cdot \nu_5 + p_{5-\mathrm{II}}^* \cdot m}{\nu_5 + m} \cdot \sigma_\mathrm{m} \quad , \quad \mathrm{kPa} \quad .$$

Relative mass flow rate at discharge:

$$\nu_6 = \frac{\nu_5 + m}{1 + m} \quad .$$

11 Exhaust nozzle (primary)

Table 11 — Input data

Symbol	Unit of measure	Description
T_8^*	K	Total temperature at entrance
p_8^*	kPa	Total pressure at entrance
ν_8	-	Relative mass flow rate at entrance
p_1	kPa	Ambient pressure
φ_n	-	Nozzle velocity loss

Nozzle pressure ratio at complete expansion:

$$\pi_{n.c}=\frac{p_8^*}{p_1}\quad .$$

Exit velocity:

$$c_9=\phi_n\sqrt{2000\cdot c_{p.cg}\cdot T_8^*\cdot\left(1-\pi_{n.c}^{-\frac{k_{cg}-1}{k_{cg}}}\right)}\quad,\quad\frac{\mathrm{m}}{\mathrm{s}}\quad .$$

Static temperature at discharge:

$$T_9=T_8^*-\frac{c_9^{\;2}}{2000\cdot c_{p.cg}}\quad,\quad\mathrm{K}\quad .$$

Relative mass flow rate at discharge:

$$\nu_9=\nu_8\quad .$$

12 Exhaust nozzle (secondary)

Table 12 — Input data

Symbol	Unit of measure	Description
$T^*_{8-\mathrm{II}}$	K	Total temperature at entrance
$p^*_{8-\mathrm{II}}$	kPa	Total pressure at entrance
p_1	kPa	Ambient pressure
φ_{nII}	-	Nozzle velocity loss

Nozzle pressure ratio at complete expansion:

$$\pi_{\mathrm{nII.c}} = \frac{p^*_{8-\mathrm{II}}}{p_1} \quad .$$

Exit velocity:

$$c_{9-\mathrm{II}} = \phi_{\mathrm{nII}} \sqrt{2000 \cdot c_{\mathrm{p}.a} \cdot T^*_{8-\mathrm{II}} \cdot \left(1 - \pi_{\mathrm{nII.c}}^{-\frac{k_a - 1}{k_a}}\right)} \quad , \quad \frac{\mathrm{m}}{\mathrm{s}} \quad .$$

Static temperature at discharge:

$$T_{9-\mathrm{II}} = T^*_{8-\mathrm{II}} - \frac{c_{9-\mathrm{II}}^{2}}{2000 \cdot c_{\mathrm{p}.a}} \quad , \quad \mathrm{K} \quad .$$

13 Features of a turbojet

Table 13 — Input data

Symbol	Unit of measure	Description
V_{f}	$\frac{\mathrm{m}}{\mathrm{s}}$	Flight velocity
c_9	$\frac{\mathrm{m}}{\mathrm{s}}$	Exit velocity
ν_9	-	Relative mass flow rate at discharge of exhaust nozzle
q_{f}	-	Relative fuel consumption
ν_4	-	Relative mass flow rate at discharge of the combustion chamber
P	kN	Thrust

Specific thrust:

$$S_P = 0{,}001 \cdot (c_9 \cdot \nu_9 - V_{\mathrm{f}}) \quad , \quad \frac{\mathrm{kN \cdot s}}{\mathrm{kg}} \quad .$$

Total air mass flow rate throw the engine:

$$G_2 = \frac{P}{S_P} \quad , \quad \frac{\mathrm{kg}}{\mathrm{s}} \quad .$$

Fuel consumption:

$$G_{\mathrm{f}} = 3600 \cdot q_{\mathrm{f}} \cdot \frac{\nu_4}{1 + q_{\mathrm{f}}} \cdot G_2 \quad , \quad \frac{\mathrm{kg}}{\mathrm{h}} \quad .$$

Specific fuel consumption:

$$S_{Gf} = \frac{G_{\mathrm{f}}}{P} \quad , \quad \frac{\mathrm{kg}}{\mathrm{kN \cdot h}} \quad .$$

14 Features of a turbofan

Table 14 — Input data

Symbol	Unit of measure	Description
V_f	$\frac{m}{s}$	Flight velocity
c_9	$\frac{m}{s}$	Exit velocity at primary exhaust nozzle
ν_9	-	Relative mass flow rate at discharge of primary exhaust nozzle
c_{9-II}	$\frac{m}{s}$	Exit velocity at secondary exhaust nozzle
m	-	Bypass ratio
q_f	-	Relative fuel consumption
ν_4	-	Relative mass flow rate at discharge of the combustion chamber
P	kN	Thrust

Specific thrust:

$$S_P=0{,}001\frac{(c_9\cdot\nu_9-V_f)+m\cdot(c_{9-II}-V_f)}{1+m}\quad,\quad\frac{kN\cdot s}{kg}\quad.$$

Total air mass flow rate throw the engine:

$$G_2=\frac{P}{S_P}\quad,\quad\frac{kg}{s}\quad.$$

Air mass flow rate throw the primary flowpath:

$$G_{2-I}=\frac{G_2}{1+m}\quad,\quad\frac{kg}{s}\quad.$$

Air mass flow rate throw the secondary flowpath:

$$G_{2-II}=m\cdot G_{2-I}\quad,\quad\frac{kg}{s}\quad.$$

Fuel consumption:

$$G_f=3600\cdot q_f\cdot\frac{\nu_4}{1+q_f}\cdot G_{2-I}\quad,\quad\frac{kg}{h}\quad.$$

Specific fuel consumption:

$$S_{Gf}=\frac{G_f}{P}\quad,\quad\frac{kg}{kN\cdot h}\quad.$$

15 Features of a turbofan with mixing

Table 15 — Input data

Symbol	Unit of measure	Description
V_{f}	$\frac{\mathrm{m}}{\mathrm{s}}$	Flight velocity
c_9	$\frac{\mathrm{m}}{\mathrm{s}}$	Exit velocity at primary exhaust nozzle
ν_9	-	Relative mass flow rate at discharge of primary exhaust nozzle
m	-	Bypass ratio
q_{f}	-	Relative fuel consumption
ν_4	-	Relative mass flow rate at discharge of the combustion chamber
P	kN	Thrust

Specific thrust:

$$S_P = 0{,}001 \cdot (c_9 \cdot \nu_9 - V_{\mathrm{f}}) \quad , \quad \frac{\mathrm{kN \cdot s}}{\mathrm{kg}} \quad .$$

Total air mass flow rate throw the engine:

$$G_2 = \frac{P}{S_P} \quad , \quad \frac{\mathrm{kg}}{\mathrm{s}} \quad .$$

Air mass flow rate throw the primary flowpath:

$$G_{2-\mathrm{I}} = \frac{G_2}{1+m} \quad , \quad \frac{\mathrm{kg}}{\mathrm{s}} \quad .$$

Air mass flow rate throw the secondary flowpath:

$$G_{2-\mathrm{II}} = m \cdot G_{2-\mathrm{I}} \quad , \quad \frac{\mathrm{kg}}{\mathrm{s}} \quad .$$

Fuel consumption:

$$G_{\mathrm{f}} = 3600 \cdot q_{\mathrm{f}} \cdot \frac{\nu_4}{1+q_{\mathrm{f}}} \cdot G_{2-\mathrm{I}} \quad , \quad \frac{\mathrm{kg}}{\mathrm{h}} \quad .$$

Specific fuel consumption:

$$S_{Gf} = \frac{G_{\mathrm{f}}}{P} \quad , \quad \frac{\mathrm{kg}}{\mathrm{kN \cdot h}} \quad .$$